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APPLICATION OF PHOTOELASTIC COATINGS TO STRAIN INVESTIGATION IN POLYCRYSTAL MICRODOMAINS

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A thin layer (coating) of optically active material is deposited on the specimen surface in investigations by the method of photoelastic coatings. If the coating thickness is relatively small, processing the experiment results offers no special difficulties since the optical quantities obtained during the experiment will be proportional to the measured strains on the specimen surface [1]. If the coating thickness is commensurate with the characteristic dimension of the strain zone, then analysis of the measurement results is complicated substantially and requires special processing methods (refinements).

This problem is especially complex for strain investigations in microdomains of real polycrystalline materials. Here even for small loads (in the domain of the so-called microplastic strains) inelastic deformation occurs by the formation and development of displacements governed by the localization of slip traces. Such zones of local strain concentration have, in turn, a finer structure and can reflect the result of the action of several strain mechanisms in the slip band domain [2, 3]. In all these cases the minimal coating thickness realizable in practice exceeds the size of the section deformed and the measurement results cannot therefore be used without appropriate correction.

Different cases of strain measurement in a slip band domain of width 2a (Fig. 1) are considered in this paper. The thickness d of the photoelastic coating being used (not shown in Fig. 1) considerably exceeds the deformation zone dimension ($d > 2a$). We hence consider the strains homogeneous in the slip band domain while they can be neglected outside this zone. We denote the projections of the displacement vector P_0 characterizing the displacement of the undeformed sections as a rigid whole on the x, y, z axes by U_0, V_0, W_0. We consider the displacements U, V, W within the deformed zone $-a \leqslant x \leqslant a$ linear functions of the coordinate x.

1. Out of all the displacement vector component, let just the vertical component V_0 be different from zero. If the length of the slip band is large in the z axis direction, it can be considered that the coating deposited on the surface y = 0 (see Fig. 1) is under plane strain conditions with the following boundary conditions:

$$\text{for } y = d \ \sigma_x = 0,\ \tau_{xy} = 0, \tag{1.1}$$

$$\text{for } y=0 \quad V = \begin{cases} -V_0, & \text{if } -\infty < x < -a, \\ V_0 x/a, & \text{if } -a \leqslant x \leqslant a, \\ V_0, & \text{if } a < x < \infty \end{cases}$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 146-150, May-June, 1984. Original article submitted October 10, 1982.

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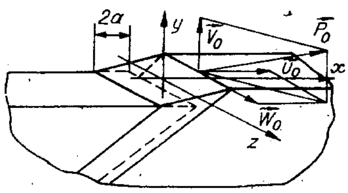

Fig. 1

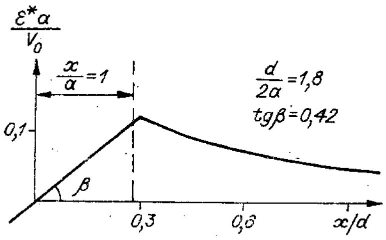

Fig. 2

(σ_x, τ_{xy} are stress tensor components for the coating).

Under such boundary conditions the stress state of a coating having a sufficiently large length along the x axis is described well by a stress function in the form of a Fourier integral [4]:

$$\varphi(x, y) = \int_{-\infty}^{\infty} F(\alpha y) \sin(\alpha x)\, d\alpha, \tag{1.2}$$

where $F(\alpha y) = A \operatorname{ch}(\alpha y) + B \operatorname{sh}(\alpha y) + C(\alpha y) \operatorname{ch}(\alpha y) + D(\alpha y) \operatorname{sh}(\alpha y)$. The constants A, B, C, D in (1.2) are found from the boundary conditions (1.1). Afterwards, the strain component can be determined for the coating fixed by the method of photoelastic coatings:

$$\varepsilon_x = \frac{1}{2G} \int_{-\infty}^{\infty} \frac{\alpha^2}{\Delta} [\Delta_1 ((3-4\nu) \operatorname{sh}(\alpha y) + (\alpha y) \operatorname{ch}(\alpha y)) + \Delta_2 (\alpha y) \operatorname{sh}(\alpha y) - H_1 \Delta \operatorname{sh}(\alpha y)] \sin(\alpha x)\, d\alpha, \tag{1.3}$$

where $H_1 = 2GV_0 \sin(\alpha a)/(\pi \alpha a^3)$; $\Delta_1 = H_1[2(1-\nu) + \operatorname{sh}^2(\alpha d)]$; $\Delta_2 = H_1[\alpha d - 0.5 \operatorname{sh}(\alpha d)]$; $\Delta = (\alpha d)^2 + 4(1-\nu)^2 + (3-4\nu)\operatorname{sh}^2(\alpha d)$; ν, G are elastic constants of the coating material.

Upon exposing the coating along the y axis, a quantity equal to the mean value of the difference between the principal strains along the coating thickness will be measured

$$\varepsilon^* = \varepsilon_1 - \varepsilon_2 = \frac{1}{d} \int_0^d \varepsilon_x dy. \tag{1.4}$$

The directions of the principal strains will evidently agree with the x and y axes in this case.

Presented as an illustration in Fig. 2 is the distribution of the measured strains along the x axis, obtained by a numerical realization of (1.3) and (1.4), when the coating thickness is 1.8 times greater than the strain zone on the specimen surface. The ratio of the strain measured ε^* to the actual value on the specimen surface is plotted along the ordinate axis. The dashed line marks off the boundary of the deformed zone. As is seen, the slope of the curve at the origin β will characterize the magnitude of the displacement V_0 for a known coating thickness and deformed zone width.

2. Let us now examine the case when only the horizontal component of the displacement vector U_0 is different from zero. The boundary conditions for the coating are

$$\text{for } y = d \;\; \sigma_y = 0,\; \tau_{xy} = 0,$$

$$\text{for } y = 0 \;\; U = \begin{cases} -U_0, & \text{if } -\infty < x < -a, \\ \dfrac{U_0}{a} x, & \text{if } -a \leqslant x \leqslant a, \\ U_0, & \text{if } a < x < \infty. \end{cases}$$

As in Sec. 1, the solution can be obtained in the form of a Fourier integral

$$\varepsilon_x = \frac{1}{2G} \int_{-\infty}^{\infty} \frac{\alpha^2}{\Delta} [\Delta_3 ((3-4\nu) \operatorname{sh}(\alpha y) + (\alpha y) \operatorname{ch}(\alpha y)) + \\ + \Delta_4 (\alpha y) \operatorname{sh}(\alpha y) + H_2 \Delta \operatorname{ch}(\alpha y)] \cos(\alpha x)\, d\alpha, \tag{2.1}$$

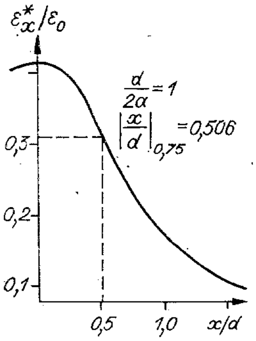

Fig. 3

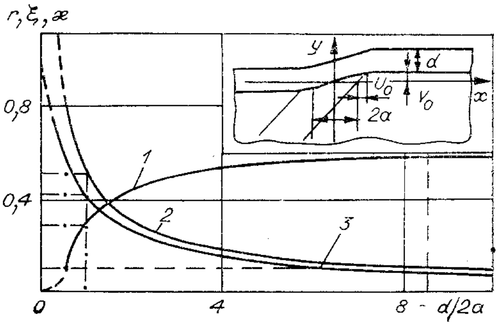

Fig. 4

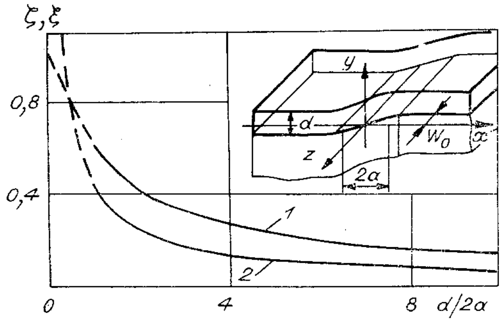

Fig. 5

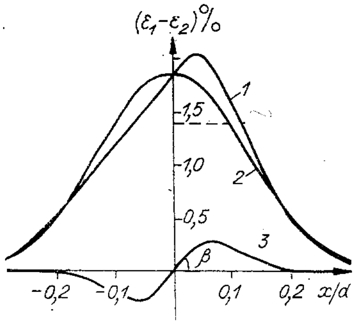

Fig. 6

$$H_2 = 2GU_0 \sin(\alpha a)/(\pi \alpha a^3), \quad \Delta_3 = -H_2[\alpha d + 0.5\,\mathrm{sh}(2\alpha d)], \qquad (2.1)$$

$$\Delta_4 = H_2[2(1-\nu) + \mathrm{sh}^2(\alpha d)].$$

We again use (1.4) to compute the difference in the principal strains averaged with respect to the coating thickness.

The distribution of strains measured upon exposure of the coating at different distances from the center of the deformed section, as obtained by computations using (1.4) and (2.1), is shown in Fig. 3. Plotted along the abscissa axis is the quantity $\varepsilon^*/\varepsilon_0$, where $\varepsilon_0 = U_0/a$ is the strain on the specimen surface in the slip band zone. The coating thickness here equals the width of the strain section. As is seen, the deformations measured at the center of the strain secion ε^*_{max} are just 43% of the actual values on the surface being investigated. Let us note that the strain measured diminished to 75% of its maximal value at the distance $x_{0.75}$ = 0.506d from the center of the strain section. This distance will be different for any other $d/2a$ relationship, which can be used to determine the actual width of the slip band.

To do this, it is necessary to construct graphs of the change in the functions $\varkappa = \varepsilon^*_{max}/\varepsilon_0$, $\xi = x_{0.75}/d$, $r = a \tan\beta/V_0$ of the argument $d/2a$ represented in Fig. 4. For instance, let the curve $\varepsilon^* = \varepsilon^*(x/d)$ be obtained experimentally. Let us note that the coating thickness is considered known, and for definiteness, we take d = 0.1 mm. From this curve we determine ξ = 0.506, ε^*_{max} = 5%. Then from the curve 2 (the dash-dot line) we find $d/2a$ = 1.0 from which $2a = d/1.0 = 0.1$. Now, the value of the displacement U_0 on the specimen surface can be determined for which we find the quantity $\varkappa = 0.42$ by using the curve 3 for the obtained value of $d/2a$; then $\varepsilon_0 = \varepsilon^*_{max}/\varkappa = 0.12$, $U_0 = \varepsilon_0 a = 0.006$ mm. If the width of the perturbation zone is known, it is possible, by determining the quantity r from curve 1, to find the value of the vertical displacement V_0 in addition.

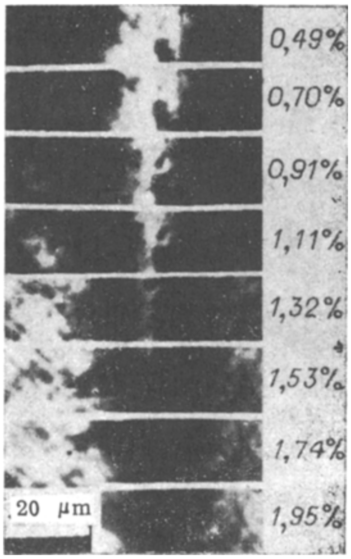

Fig. 7

3. Finally, let us consider the case when only the component along the z axis out of all the displacement vector components is not zero on the surface under investigation. The equilibrium equation for the coating can here be reduced to the following

$$\partial\tau_{xz}/\partial x + \partial\tau_{yz}/\partial y = 0, \tag{3.1}$$

which corresponds formally to the equilibrium equation for the plane problem. We hence use the method applied above for its solution.

The boundary conditions are

$$\text{at } y = d \ \sigma_y = 0,\ \tau_{xy} = 0,$$
$$\text{at } y = 0 \ W = \begin{cases} -W_0, & \text{if } -\infty < x < -a, \\ \dfrac{W_0}{a}x, & \text{if } -a \leqslant x \leqslant a, \\ W_0, & \text{if } a < x < \infty. \end{cases} \tag{3.2}$$

Solving (3.1) with (3.2) taken into account, we find the strain in the zOx plane of the coating

$$\gamma_{xz} = \frac{W_0}{2\pi}\int\limits_{-\infty}^{\infty}\frac{\sin(\alpha a)}{\alpha a}[\operatorname{ch}(\alpha y) - \operatorname{th}(\alpha d)\operatorname{sh}(\alpha y)]\,d\alpha.$$

Upon exposing the coating, the mean value of the difference in the principal strains is measured along the y axis, which can be found as before by numerical integration

$$\varepsilon_1 - \varepsilon_2 = \gamma^* = \frac{1}{d}\int\limits_0^d \gamma_{xz}dy.$$

However, the direction of the principal strains differs here from the preceding cases by making a 45° angle with the direction of the strain section. This circumstance permits easily separating the two cases considered earlier from this last one. The measured strain γ^* reaches its greatest value at the center of the strained section (x = 0) and depends substantially on the coating thickness. The change in the strain $\zeta = \gamma^*/\gamma_0$ at the center of the strained zone is shown by curve 1 in Fig. 5 as a function of the relative thickness of the coating, while curve 2 characterizes the distance from the center at which the measured strain is diminished to 75% for different coating thicknesses. This permits determination of the size of the deformed zone by the picture of the measured strains, and then by using curve 1 finding the real value of the strains at the lower edge of the coating, which is in agreement with the actual strain in the domain under investigation.

4. Furthermore, let us consider the possibility of determining the displacement on the surface under investigation in cases when U_0 and V_0 are not zero. We use the circumstance that the quantities measured by the method of photoelastic coatings are distributed skew-symmetrically for $V_0 \neq 0$ (Sec. 1) and symmetrically for $U_0 \neq 0$ (Sec. 2) relative to the origin (see Fig. 3) to determine them separately. Moreover, we note that the symmetric curve decreases monotonically on the section $(0, \infty)$ by approaching zero asymptotically, while the skew-symmetric curve in the same interval has one extremum and also approaches zero asymptotically at infinity.

When $U_0 \neq 0$ and $V_0 \neq 0$, a curve of general form (curve 1 in Fig. 6) is determined by the method of photoelastic coatings, which can be separated into symmetric and skew-symmetric distributions by using the condition formulated above. Now, all the parameters governing the slip band can be determined from Fig. 4.

As an illustration, let us present the results of investigating the strain in the slip band domain of a specimen of magnesium alloy after unloading. An 80-μm-thick coating of epoxy compound was deposited on the specimen prior to loading.

Presented in Fig. 7 are photographs of the band pattern for the very same section of the specimen oriented so that the long side of each photograph is perpendicular to the slip band direction. The photographs in Fig. 7 differ by the fact that they are obtained for different values of the optical difference in the compensator, and therefore yield interference fringes of fractional order [1]. The appropriate difference in the principal strains (indicated to the right on the photographs) was then determined for each interference fringe. From the results of processing these patterns, a distribution of the uncorrelated differences in the principal strains ε^* (see Fig. 6) was determined along the normal to the slip band direction. The location of the center of the strained zone is found computationally by sampling the best compliance with the conditions formulated above. Afterwards, the curve 1 could be considered as the superposition of the symmetric and skew-symmetric components (curves 2 and 3, respectively). Now, by using the graphs of Fig. 4, the slip band parameters can be determined by the following scheme: $\varepsilon^*_{max} = 1.84\%$, $0.75\varepsilon^*_{max} = 1.38\%$, $\xi = 0.1$ (dashed line in Fig. 6); then (see Fig. 4), $d/2a = 8.5$, $2a = 9.41$ μm ($d = 80$ μm), $\varkappa = \varepsilon^*_{max}/\varepsilon_0 = 0.8$, $\varepsilon_0 = 23\%$, $2U_0 = 2.16$ μm. From curve 1 in Fig. 4 we determine the quantities $r = 0.48$, $\tan\beta = 0.84$, $2V_0 = 16.47$ μm.

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